Recent Developments in Vorton Theory

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This article provides a concise overview of recent theoretical results concerning the theory of vortons, which are defined to be (centrifugally supported) equilibrium configurations of (current-carrying) cosmic string loops. Following a presentation of the results of work on the dynamical evolution of small circular string loops, whose minimum energy states are the simplest examples of vortons, recent order-of-magnitude estimates of the cosmological density of vortons produced in various kinds of theoretical scenarios are briefly summarized.

1. INTRODUCTION

It is rather generally accepted (Shellard and Vilenkin, 1994) that among the conceivable varieties of topological defects of the vacuum that might have been generated in early phase transitions, the *vortex-type* defects describable on a macroscopic scale as *cosmic strings* are the kind that is most likely to actually occur—at least in the postinflationary epoch—because the other main categories, namely monopoles and walls, would produce a catastrophic cosmological mass excess. Even a single wall stretching across a Hubble radius would by itself be too much, while in the case of monopoles it is their collective density that would be too high unless the relevant phase transition occurred at an energy far below that of the GUT level, a possibility that is commonly neglected on the grounds that no monopole formation occurs in the usual models for the transitions in the relevant range, of which the most important is that of electroweak symmetry breaking.

The case of cosmic strings is different. One reason is that (although they are not produced in the standard electroweak model) strings are actually produced at the electroweak level in many of the commonly considered (e.g., supersymmetric) alternative models. A more commonly quoted reason why

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the case of strings should be different, even if they were formed at the GUT level, is that—while it may have an important effect in the short run as a seed for galaxy formation—such a string cannot be cosmologically dangerous just by itself, while a distribution of cosmic strings is also cosmologically harmless because (unlike "local" as opposed to "global" monopoles) they will ultimately radiate away their energy and progressively disappear. However, while this latter consideration is indeed valid in the case of ordinary Goto–Nambu-type strings, it was pointed out by Davis and Shellard (1988, 1989) that it need not apply to "superconducting" current-carrying strings of the kind originally introduced by Witten (1985). This is because the occurrence of stable currents allows loops of string to be stabilized in states known as "vortons," so that they cease to radiate.

The way this happens is that the current, whether timelike or spacelike, breaks the Lorentz invariance along the string worldsheet (Carter, 1989a, b, 1995a; Peter, 1992a, b), thereby leading to the possibility of rotation, with velocity v say. The centrifugal effect of this rotation may then compensate the string tension T in such a way as to produce an equilibrium configuration, i.e., what is known as a *vorton*, in which

$$T = v^2 U \tag{1}$$

where U is the energy per unit length in the corotating rest frame (Carter, 1990, 1995a). This condition is interpretable as meaning that the circulation velocity v is the same as the velocity (Carter, 1989a, b) of extrinsic (transverse) "wiggle"-type perturbation—so that relatively backward-moving "wiggles" will be effectively static deformations. Such a vorton state will be stable, at least classically, if it minimizes the energy for given values of the pair of conserved quantities characterizing the current in the loop, namely the phase winding number N, say, and the corresponding particle number Z, say, whose product

$$J = NZ \tag{2}$$

is interpretable in the case of a circular loop as the magnitude of its angular momentum. If the current is electromagnetically coupled with charge coupling constant e, then the latter will determine a corresponding vorton charge $Q = Ze$.

Whereas the collective energy density of a distribution of nonconducting cosmic strings will decay in a similar manner to that of a radiation gas, in contrast, for a distribution of relic vortons, the energy density will scale like that of ordinary matter. Thus, depending on when and how efficiently they were formed, and on how stable they are in the long run, such a distribution of vortons might eventually come to dominate the density of the universe. It has been rigorously established (Carter, 1993; Martin, 1994, 1995) that

circular vorton configurations of this kind will commonly (though not always) be stable in the dynamic sense at the classical level, but very little is known so far about noncircular configurations or about the question of stability against quantum tunneling effects, one of the difficulties being that the latter is likely to be sensitively model dependent.

2. DYNAMICS OF CIRCULAR STRING LOOPS

Using the kind of string model (Carter and Peter, 1995) that is appropriate for describing the effect of a Witten-type current, Larsen and Axenides (1997) have recently carried out an analytic treatment of the special case of a free circular string loop for which the angular momentum J vanishes, so that there is no centrifugal effect to prevent the ultimate collapse of the loop. Extending this to the case for which a centrifugal is present barrier, we have undertaken a systematic analysis (Carter *et al.*, 1997) of the dynamics of a free circular string loop carrying a Witten-type current in the generic case for which both N and Z have nonzero values. These values will be given in terms of a pair of Bernoulli-type constants of integration, B and C say, by

$$N = \frac{B}{2\pi\sqrt{\kappa_0}}, \qquad Z = C\sqrt{\kappa_0} \qquad (3)$$

where κ_0 is a constant of the order of unity that depends on the particular form of the string model. The kind of string model needed for representing the macroscopic effect of a Witten-type "superconducting" vacuum vortex is specified (Carter and Peter, 1995) by a Lagrangian function $\mathcal{L}$ depending on a single scalar variable w that is proportional to the squared magnitude of the scalar phase variable (whose loop integral specifies the winding number N) with κ_0 as the proportionality constant. An important role in the analysis is played by the derived function $\mathcal{K}$ that is obtainable from the Lagrangian using the formula

$$\mathcal{K} = -\left(2\frac{d\mathcal{L}}{dw}\right)^{-1} \qquad (4)$$

and that is adjusted (by the convention used to fix the normalization of κ_0) so that $\mathcal{K}$ tends to unity in the null current limit, i.e., as w tends to zero.

The solutions fall generically into two distinct classes, namely those for which $B^2 > C^2$, in which the string current remains always spacelike, i.e., $w > 0$, and those for which $B^2 < C^2$, in which the current remains always timelike, i.e., $w < 0$. Intermediate between these classes is the special "chiral" limit case characterized by $B^2 = C^2$, in which the current remains always

null, i.e., $w = 0$. A key intermediate step in the analysis is the establishment of a relation of the form

$$\ell^2 = \frac{B^2 - C^2 \mathcal{H}^2}{w} \tag{5}$$

which determines the loop circumference $\ell = 2\pi r$ as a function of w and thus implicitly determines the state variable w as a function of ℓ, except in the special "chiral" limit case $B^2 = C^2$, for which $w = 0$, so that the right-hand side is indeterminate. The outcome of the analysis (Carter et al., 1997) is that the string radius r is governed by an equation of the form

$$M^2 \dot{r}^2 = M^2 - \Upsilon^2 \tag{6}$$

(using a dot for time differentiation) in which M is the constant mass energy of the loop and Υ is an effective potential that is determined as a function of the loop circumference in the form

$$\Upsilon = \frac{C^2 \mathcal{H}}{\ell} - \mathcal{L}\ell \tag{7}$$

where the coefficients $\mathcal{H}$ and $\mathcal{L}$ have constant values (the former being just unity) in the "chiral" case, while in general they will be determined as (rather slowly varying) functions of ℓ by the relation (5). It is to be observed that there is an asymptotic confining potential contribution that rises linearly with the radius, and that (unlike the familiar Kepplerian particle problem, for which the centrifugal barrier goes as the inverse square of the radius) in the present case the effective centrifugal barrier goes just as the inverse first power of the radius.

It is evident in the "chiral" case, and can be verified in general, that the effective potential given by (7) will have a gradient given by

$$\frac{d\Upsilon}{d\ell} = -\frac{\mathcal{B}^2}{\mathcal{H}\ell^2} - \mathcal{L} \tag{8}$$

which vanishes at the minimum of Υ where

$$B^2 \left(\mathcal{L} + \frac{w}{\mathcal{H}} \right) = C^2 \mathcal{H}^2 \mathcal{L} \tag{9}$$

It can be seen from (6) that the value of Υ at this minimum will also be the minimum admissible value of the mass parameter for the given values of B and C, and that when M has this minimum value the loop will be in a state of equilibrium with radius given by the solution of (9), which can be shown to be equivalent to the vorton equilibrium condition (1) that was quoted at the outset.

Whether such a vorton equilibrium state is actually attainable depends on whether the minimality condition (9) actually has a solution in the admissible range of string states. The explicit form of the relevant string model (Carter and Peter, 1995), is given by

$$\mathcal{L} = -m^2 - m_*^2 \ln \sqrt{\mathcal{H}} \tag{10}$$

with

$$\mathcal{H} = 1 + \frac{w}{m_*^2} \tag{11}$$

where m and m_* are constants. The first one, m, is the relevant Kibble mass, as characterized by the condition that m^2 should be the common limit of the tension T and the energy density U in the zero-current limit. The other one, m_*, is the mass scale associated with the carrier field responsible for the Witten current. The validity of this model is limited to the range

$$e^{-2m^2/m_*^2} < \mathcal{H} < 2 \tag{12}$$

the lower bound being where the tension T tends to zero in the timelike current regime, while the upper bound is the saturation limit in the timelike current regime. It can be seen that the loop will oscillate periodically without approaching either of these "dangerous" limits provided the ratio B/C is not too far from unity (i.e., from the "chiral" value).

3. ORDER-OF-MAGNITUDE ESTIMATES

Whether or not it is exactly circular (as assumed in the analytic treatment that has just been described), the numerical value of the total mass M of a vorton state characterized by the quantum numbers N and Z will be given (Carter, 1990, 1991) in rough order of magnitude by a formula of the form

$$M \approx |NZ|^{1/2}m \tag{13}$$

where m is the relevant Kibble mass, which will normally be given approximately by the mass of the Higgs field responsible for the relevant vacuum symmetry breaking.

In the earliest crude quantitative estimates (Davis and Shellard, 1988, 1989; Carter, 1991) of the likely properties of a cosmological vorton distribution produced in this way, it was assumed not only that the current was stable against leakage by tunneling, but also that the Witten mass scale m_* characterizing the relevant carrier field was of the same order of magnitude as the Kibble mass scale m characterizing the string itself. The most significant development in the more detailed investigations carried out more recently

(Carter, 1995b, Brandenberger *et al.*, 1996) is the extension to cases in which m_* is considerably smaller than m. A rather extreme example that immediately comes to mind is that for which m is postulated to be at the GUT level, while m_* is at the electroweak level, in which case it is found that the resulting vorton density will be far too low to be cosmologically significant.

The simplest scenarios are those for which (unlike the example just quoted) the relation

$$\sqrt{m_*} \gtrsim m \tag{14}$$

is satisfied in dimensionless Planck units as a rough order-of-magnitude inequality. In this case the current condensation would have occurred during the regime in which [as pointed out by Kibble (1976, 1980) in the early years of cosmic string theory] the dynamics was dominated by friction damping. Under these circumstance it is estimated (Carter, 1995b; Brandenberger *et al.*, 1996) that the typical value of the quantum numbers of vortons in the resulting population will be given very roughly by

$$N \approx Z \approx m^{1/2} m_*^{-3/4} \tag{15}$$

which by (13) implies a typical vorton mass given by

$$M \approx \left(\frac{m}{\sqrt{m_*}}\right)^{3/2} \tag{16}$$

which, in view of (14), will never exceed the Planck mass. When the cosmological temperature has fallen to a value Θ, say, the estimated number density n of the vortons is given as a constant fraction of the corresponding number density $\approx \Theta^3$ of blackbody photons by the rough order-of-magnitude formula

$$\frac{n}{\Theta^3} \approx \left(\frac{\sqrt{m_*}}{m}\right)^3 m_*^3 \tag{17}$$

It follows in this case that, in order to avoid producing a cosmological mass excess, the value of m_* in this formula should not exceed a limit that works out to be of the order of 10^{-9}, and the limit is even smaller, around 10^{-11}, when the two scales m_* and m are comparable.

Limits in roughly the same range, round about 10^{-10} (about midway between the GUT value 10^{-3} and the electroweak value 10^{-16}) are also obtained (Brandenberger *et al.*, 1996) for m_* in scenarios for which, instead of (14),

$$\sqrt{m_*} << m \tag{18}$$

but in this case the analysis is more complicated and also more uncertain,

since the result is sensitive to the value of an efficiency factor ϵ that is expected to be of order unity, but whose exact evaluation will require numerical work that will have to be much more advanced than has been possible so far. In terms of this quantity the relevant analogue of (17) is expressible as

$$\frac{n}{\Theta^3} \approx \left(\frac{\sqrt{m_*}}{m}\right)^3 (\sqrt{m_*})^{(3+\epsilon)/(3-\epsilon)} \tag{19}$$

which means that the vorton number density n will typically be very much lower than in the preceding case, but this is compensated, as far as their contribution to the mass density of the universe is concerned, by the fact that the typical vorton mass M will be much higher: the relevant analogue of (16) is

$$M \approx \frac{m^2}{m_*} \tag{20}$$

which, in view of (18), will always exceed the Planck mass.

Even if they contribute only a negligibly small fraction of the density of the universe, the vortons may nevertheless give rise to astrophysically interesting effects: in particular, it has recently been suggested by Bonazzola and Peter (1997) that they might account for otherwise inexplicable cosmic ray events.

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